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TO FORMULATING A NONLINEAR INITIAL-BOUNDARY PROBLEM OF AN UNSTEADY SEPARATED FLOW AROUND AN AIRFOIL

D. N. Gorelov

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A general formulation of a nonlinear initial-boundary problem of an unsteady separated flow around an airfoil by an ideal incompressible fluid is considered. The problem is formulated for a complex velocity. Conditions of shedding of vortex wakes from the airfoil are analyzed in detail. The proposed system of functional relations allows constructing algorithms for solving a wide class of problems of the wing theory.

Key words: *initial-boundary problem, separated flow, airfoil, vortex wake.*

Among problems of separated flows, of special importance is a wide class of problems associated with an unsteady two-dimensional potential flow around an airfoil by an ideal incompressible fluid. Formulation of such problems and methods of their solution depend on many factors, the main ones being the flow regime (non-separated or separated) and the airfoil shape (closed or open). In addition, the flow around a smooth contour without corner points and the flow around a contour with one or several corner points should be distinguished. Intense theoretical investigations in this direction have been performed since the middle of the 20th century. Various mathematical models have been proposed (see, e.g., [1–10]), but the problem of an unsteady separated flow has not been solved yet.

The present paper deals with issues associated with formulation of a nonlinear initial-boundary problem of an unsteady two-dimensional potential flow around an airfoil. Particular attention is paid to modeling vortex wakes and analyzing the conditions of vortex-wake shedding from the airfoil.

1. FORMULATION OF THE PROBLEM

We use a Cartesian coordinate system Oxy, in which the fluid at an infinite point moves with a velocity $\boldsymbol{v}_\infty$. We assume that the fluid contains an airfoil $L(t)$ with one corner point, which starts moving at the time $t = 0$ with a velocity $\boldsymbol{U}(x, y, t)$, $(x, y) \in L(t)$. In the general case, the velocity circulation $\Gamma(t)$ around the airfoil changes with time, generating a vortex wake in a non-separated flow or vortex wakes in a separated flow around the airfoil. The vortex wakes are simulated by lines of tangential discontinuities $L_w(t)$ [or $L_{w1}(t)$ and $L_{w2}(t)$], along which the tangential component of velocity is discontinuous (Fig. 1). The fluid motion outside the airfoil and vortex wakes is assumed to be potential.

The potential character of the flow allows us to consider a complex velocity $\bar{v}(z, t) = v_x(x, y, t) - iv_y(x, y, t)$, where $z = x + iy$. The initial-boundary problem for the complex velocity can be formulated as follows. In the flow domain outside the airfoil $L(t)$ and vortex wakes $L_{wp}(t)$ $(p = 1, 2)$, the complex velocity $\bar{v}(z, t)$ is an analytical function satisfying the boundary conditions of decaying of the disturbed velocity at an infinite point and fluid non-penetration through the airfoil contour, as well as the conditions of continuity of pressure and the normal component of velocity across the vortex wakes. The flow domain where the function $\bar{v}(z, t)$ is determined is unknown beforehand and changes with time. The boundaries of the flow domain and the velocity field of the fluid are set as the initial

Omsk Department of the Institute of Mathematics, Siberian Division, Russian Academy of Sciences, Omsk 644099; gorelov@iitam.omsk.net.ru. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 48, No. 2, pp. 48–56, March–April, 2007. Original article submitted July 12, 2005; revision submitted June 15, 2006.

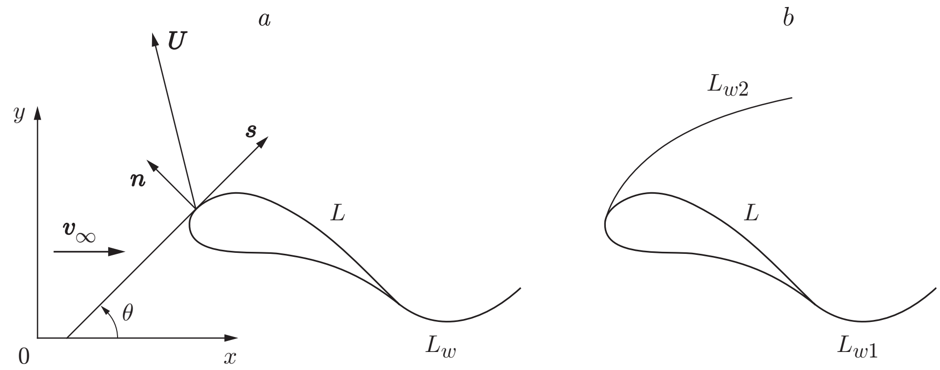

Fig. 1. Models of an unsteady flow around an airfoil: non-separated flow (a) and separated flow (b).

conditions at the time $t = 0$. For $t < 0$, it is usually assumed that the airfoil is motionless ($\boldsymbol{U} = 0$), the flow is steady, and there are no vortex wakes.

The airfoil contour and the vortex wakes are modeled by vortex layers (it is assumed that a closed contour contains the fluid moving together with the contour). The complex velocity $\bar{v}(z,t)$ is described by the expression

$$\bar{v}(z,t) = \bar{v}_\infty + \Delta\bar{v}(z,t) + \frac{1}{2\pi i}\int\limits_{L(t)} \frac{\gamma(s,t)\,ds}{z-\zeta(s,t)}, \qquad \zeta(s,t) \in L(t),$$
$$\Delta\bar{v}(z,t) = \sum_{p=1}^{N_w} \frac{1}{2\pi i}\int\limits_{L_{wp}(t)} \frac{\gamma_{wp}(\sigma,t)\,d\sigma}{z-\zeta_{wp}(\sigma,t)}, \qquad \zeta_{wp}(\sigma,t) \in L_{wp}(t). \tag{1.1}$$

Here $\Delta\bar{v}(z,t)$ is the complex velocity induced by vortex wakes $L_{wp}(t)$ shed from the contour $L(t)$, N_w and $\gamma_{wp}(\sigma,t)$ are the number of vortex wakes and their intensity, respectively, $\gamma(s,t)$ is the intensity of the vortex layer modeling the contour $L(t)$, and s and σ are the arc coordinates. Integration over the contour $L(t)$ in Eqs. (1.1) is performed in the clockwise direction, and the arc coordinate σ on the vortex wakes is counted from the point of vortex shedding from the contour $L(t)$ along each vortex wake $L_{wp}(t)$.

Equations (1.1) satisfy the condition of decaying of the disturbed velocity at an infinite point, because $\bar{v}(z,t) = \bar{v}_\infty$ as $|z| \to \infty$. The requirement that the condition of fluid non-penetration through the contour $L(t)$ should be satisfied yields two singular integral equations of the second and first kind, respectively [10]:

$$\frac{1}{2}\,\gamma(s,t) + \mathrm{Re}\Big[\mathrm{e}^{i\theta(z,t)}\Big(\bar{v}_\infty + \Delta\bar{v}(z,t) + \frac{1}{2\pi i}\int\limits_{L(t)} \frac{\gamma(s,t)\,ds}{z-\zeta(s,t)} - \bar{U}(z,t)\Big)\Big] = 0, \quad z \in L(t); \tag{1.2}$$

$$\mathrm{Im}\Big[\mathrm{e}^{i\theta(z,t)}\Big(\bar{v}_\infty + \Delta\bar{v}(z,t) + \frac{1}{2\pi i}\int\limits_{L(t)} \frac{\gamma(s,t)\,ds}{z-\zeta(s,t)} - \bar{U}(z,t)\Big)\Big] = 0, \qquad z \in L(t). \tag{1.3}$$

Equations (1.2) and (1.3) may be solved independently. The sought function in these equations is the vortex-layer intensity $\gamma(s,t)$ on the contour $L(t)$. The vortex-wake intensities $\gamma_{wp}(\sigma,t)$ and the wakes $L_{wp}(t)$ that enter Eqs. (1.1) are related to $\gamma(s,t)$ by additional expressions.

Concerning the boundary conditions on vortex wakes, they are satisfied if the vortex wakes move freely with the fluid. This occurs if the complex coordinate $z_{wp}(\sigma,t)$ of the vortex shed from the point z_{*p} of the airfoil contour at a certain time τ ($0 \le \tau < t$) is found by solving the nonlinear differential equation

$$\frac{d\bar{z}_{wp}}{dt} = \bar{v}_0(z_{wp},t) \tag{1.4}$$

with the initial condition $\bar{z}_{wp}(0,\tau) = \bar{z}_{*p}(\tau)$, where $\tau \in [0,t)$, and a prescribed velocity field at the time τ. Here τ is a Lagrangian coordinate of the vortex considered and $\bar{v}_0(z_w,t)$ is the complex velocity of vortices in the wake, which is equal to the halved sum of the limiting values of the complex velocity $\bar{v}(z,t)$ in approaching the point $z_{wp} \in L_{wp}$.

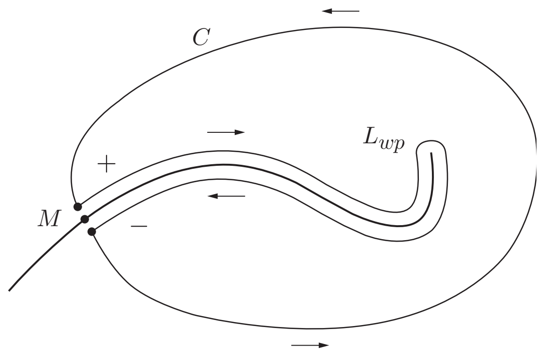

Fig. 2. Schematic for determining the velocity-potential discontinuity at the point M of the vortex wake.

Equation (1.4) is nonlinear, and its solution depends on the entire history of airfoil motion. These are the specific features and difficulties of the problem considered. Problems of this type are currently solved with the use of discretization in time [9], which allows one to reduce the original problem with time-dependent unknown boundaries to consecutive solution of boundary-value problems in fixed domains for several times $t_1, \ldots, t_n$. Thus, the initial-boundary problem reduces to solving two problems. One of them is the Cauchy problem associated with solving the nonlinear differential equation (1.4) for the time t_n with the initial conditions set at the time t_{n-1}. The other problem is the boundary-value problem for the time t_n under consideration. In this case, all boundaries of the flow domain are assumed to be known. The boundary-value problem reduces to solving one of the linear integral equations (1.2) or (1.3) for a closed contour or to solving the integral equation of the first kind (1.3) for an open contour (arc). Equations (1.2) and (1.3) are normally solved by the method of discrete vortices or by the panel method [9]. A comparative analysis shows that the panel method for closed contours predicts more reliable results, in particular, in solving the integral equation of the second kind [11].

2. RELATIONS ON THE WAVE VORTEX

Let us derive additional relations between the characteristics of the vortex wakes with the intensity of the vortex layer on the airfoil contour $L(t)$. First, we have to determine the discontinuity of the velocity potential $\varphi(x, y, t)$ at the transition across the vortex wake. We denote the limiting values of the velocity potential in approaching the point $M \in L_{wp}$ by $\varphi_+(M, t)$ and $\varphi_-(M, t)$. We consider an arbitrary contour C, which covers the vortex wake L_{wp} and crosses the latter one time at a point M (Fig. 2). Velocity circulation over the contour C is $\Gamma_C = \varphi_+(M, t) - \varphi_-(M, t)$. Let us consider a closed contour composed of the contour C and the edges of the cut made along the vortex wake L_{wp}. This contour does not cross the vortex wake, and velocity circulation over this contour equals zero. The fluid velocities at the cut edges are $v_\sigma^+(\sigma, t)$ and $v_\sigma^-(\sigma, t)$, respectively, and the difference in these velocities determines the vortex-wake intensity:

$$\gamma_{wp}(\sigma, t) = v_\sigma^-(\sigma, t) - v_\sigma^+(\sigma, t). \tag{2.1}$$

With allowance for Eq. (2.1), the velocity-potential discontinuity at the point $M \in L_{wp}$ is determined by the formula

$$\varphi_+(M, t) - \varphi_-(M, t) = \int\limits_{\sigma_M}^{l_{wp}} \gamma_{wp}(\sigma, t)\, d\sigma, \tag{2.2}$$

where σ_M is the arc coordinate of the point M and l_{wp} is the length of the vortex wake L_{wp} at the time t. To obtain another important relation on the vortex wake, we use the condition of continuity of the pressure $p(x, y, t)$ across L_{wp}. The pressure is determined by the Cauchy–Lagrange integral, which can be transformed to the following form at the points of the vortex wake L_{wp} [12]:

$$p = -\rho\Big(\frac{\delta\varphi}{\delta t} + \frac{1}{2}\,(v_\sigma - v_{e\sigma})^2 - \frac{1}{2}\,v_e^2\Big) + C(t). \tag{2.3}$$

Here the operator $\delta/\delta t$ determines the derivative with respect to time t at the point $M \in L_{wp}$ moving with the transport velocity $\boldsymbol{v}_e$; $C(t)$ is an arbitrary function of time. It follows from Eqs. (2.1) and (2.3) that the pressure difference at the points of the vortex wake is

$$p_+ - p_- = -\rho\Big(\frac{\delta}{\delta t}(\varphi_+ - \varphi_-) + \frac{1}{2}\Big[(v_\sigma^+ - v_{e\sigma})^2 - (v_\sigma^- - v_{es})^2\Big]\Big)$$

$$= -\rho\Big(\frac{\delta}{\delta t}(\varphi_+ - \varphi_-) + \gamma_{wp}(v_{e\sigma} - v_{0\sigma})\Big), \qquad v_{0\sigma} = \frac{v_\sigma^+ + v_\sigma^-}{2}. \tag{2.4}$$

Let the point M move along the vortex wake L_{wp} with a velocity $v_{0\sigma}$. In this case, the transport velocity of the point M is $v_{e\sigma} = v_{0\sigma}$. Then, in accordance with Eqs. (2.2) and (2.4), the condition of pressure continuity across the vortex wake L_{wp} is satisfied if

$$\frac{\delta}{\delta t}[\varphi_+(M,t) - \varphi_-(M,t)] = \frac{d}{dt}\int_{\sigma_M}^{l_{wp}} \gamma_{wp}(\sigma,t)\,d\sigma = 0. \tag{2.5}$$

It should be noted that the velocity $v_{0\sigma}$ at the point $M \in L_{wp}$ is determined by Eqs. (1.1) where the corresponding singular integral is understood in the sense of the Cauchy principal value. It is with this velocity that the vortices move along the vortex wake, which is described by Eq. (1.4).

Let us consider Eq. (2.5) for two arbitrary points $M_1 \in L_{wp}$ and $M_2 \in L_{wp}$ moving together with the fluid along the vortex wake. Subtracting one expression from the other, we obtain

$$\int_{\sigma_1}^{\sigma_2} \gamma_{wp}(\sigma,t)\,d\sigma = \text{const}, \tag{2.6}$$

where $\sigma_1(t)$ and $\sigma_2(t)$ are the arc coordinates of the points M_1 and M_2. Equation (2.6) implies that the total intensity of an arbitrary element of the vortex wake whose points move together with the fluid remains constant during the entire time of its motion. The element itself is deformed thereby; its length and vortex intensity $\gamma_{wp}(\sigma,t)$, $\sigma_1(t) < \sigma < \sigma_2(t)$ become different.

Equation (2.6) forms the basis for mathematical modeling of vortex wakes by a system of free discrete vortices shed from an airfoil during a certain small time interval. Such a discrete model of the vortex wake is widely used in calculations, starting from [1].

Equation (2.6) allows us to conclude that the intensity of the vortex wake L_{wp} is determined by the intensity of this wake at the point of its shedding from the contour $L(t)$.

3. CONDITIONS AT THE POINTS OF VORTEX-WAKE SHEDDING FROM THE CONTOUR

3.1. General Condition. The vortex wakes $L_{wp}(t)$ are supplemented by vortices shed from the contour at certain points $z_{*p}(t)$ with the arc coordinates $s_{*p}(t)$. These points may be fixed on the contour $L(t)$ and move together with the contour with a complex velocity $\bar{U}(z_{*p},t)$ or move along the contour with a certain velocity $ds_{*p}(t)/dt$. Fixed points are usually located at the sharp or angular edge on the contour, whereas moving points are located on smooth portions of the contour $L(t)$. In the wake proper, the vortices move with a complex velocity $\bar{v}_0(z_{wp},t)$ equal to the halved sum of the limiting values of the fluid velocity on different sides of the wake. The vortex-shedding velocity is determined by the relative velocity, which is the difference between the absolute velocity of vortices in the wake [at the point $z_{*p}(t)$] and the transport velocity of the shedding point. Vortex shedding occurs tangentially to the contour; therefore, the vortex-shedding velocity $w_p(t)$ from the contour $L(t)$ to the wake $L_{wp}(t)$ is determined by the expression

$$w_p(t) = v_{0\sigma}(s_{*p},t) - v_{e\sigma}(s_{*p},t) \qquad (p = 1,\dots,N_w), \tag{3.1}$$

where $v_{e\sigma}(s_{*p},t)$ is the transport velocity of the shedding point equal to the absolute velocity of this point in the original Cartesian coordinate system. It should be noted that vortex shedding from the contour is possible only for $w_p(t) > 0$.

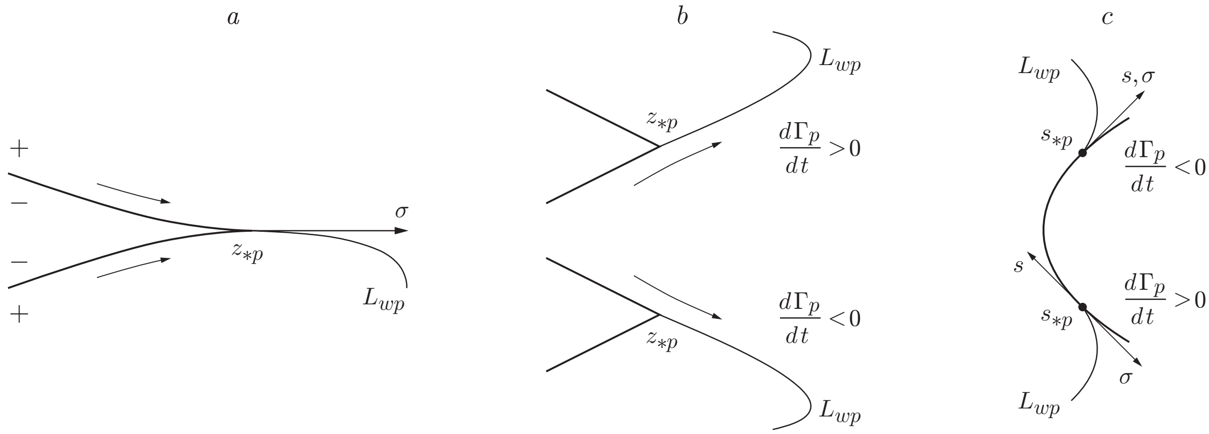

Fig. 3. Shedding of the wave vortex from a sharp edge (a), from an angular edge (b), and from a point on a smooth contour (c).

Let us derive a general expression relating the intensity of the shed vortices $\gamma_{wp}(0,t)$, velocity of their shedding $w_p(t)$, and velocity circulation $\Gamma_p(t)$ around the vortex wake $L_{wp}(t)$. For this purpose, we consider Eq. (2.4) assuming that the point $M \to z_{*p}$ from the vortex-wake boundary ($\sigma_M \to 0$). Setting the pressure-continuity condition at this point and using Eqs. (2.2) and (3.1), we obtain the general expression at the point of vortex shedding from the contour

$$\frac{d}{dt}\,\Gamma_p(t) = \gamma_{wp}(0,t)w_p(t), \qquad p = 1,\ldots,N_w.$$

Using Kelvin's theorem on a constant velocity circulation around a closed fluid contour, we can write another expression

$$\frac{d}{dt}\,\Gamma(t) + \sum_{p=1}^{N_w} \frac{d}{dt}\,\Gamma_p(t) = 0, \qquad \Gamma(t) = \int\limits_{L(t)} \gamma(s,t)\,ds,$$

which relates the shedding velocities and intensities of the shed vortex wakes to velocity circulation over the contour $L(t)$.

Let us consider the conditions of vortex shedding to the wake for shedding points of different types in more detail.

3.2. Sharp Edge. On a sharp edge (at the cuspidal point), the tangential line to the vortex wake $L_{wp}(t)$ coincides with the tangential line to the contour $L(t)$ ahead of the edge and is directed opposite to the latter behind the edge. Therefore, $v_\sigma^+(0,t) = v_s^+(s_{*p} - 0, t)$, and $v_\sigma^-(0,t) = -v_s^+(s_{*p} + 0, t)$. From Eq. (2.1) and $\gamma(s,t) = v_s^-(s,t) - v_s^+(s,t)$, it follows that the intensity of the shed vortices is found as

$$\gamma_{wp}(0,t) = \gamma(s_{*p} - 0, t) + \gamma(s_{*p} + 0, t). \qquad (3.2)$$

Similarly, we obtain the expression for the absolute velocity of vortices entering the wake:

$$v_{0\sigma}(s_{*p},t) = \frac{v_s^+(s_{*p} - 0, t) - v_s^+(s_{*p} + 0, t)}{2} = U_\sigma(s_{*p},t) - \frac{\gamma(s_{*p} - 0, t) - \gamma(s_{*p} + 0, t)}{2}. \qquad (3.3)$$

The transport velocity of the shedding point is determined as $v_{e\sigma}(s_{*p},t) = U_\sigma(s_{*p},t)$. Hence, the velocity of vortex shedding from a sharp edge is

$$w_p(t) = v_{0\sigma}(s_{*p},t) - v_{e\sigma}(s_{*p},t) = -[\gamma(s_{*p} - 0, t) - \gamma(s_{*p} + 0, t)]/2. \qquad (3.4)$$

3.3. Angular Edge. In the case of an angular edge, vortex shedding occurs tangentially to one of the contour sides, depending on the sign of the derivative $d\Gamma_p(t)/dt$ (Fig. 3).

For an angular edge, relations (3.2)–(3.4) remain unchanged. In the vicinity of the corner point, one of the streamline emanating from the contour has an inflection, and the relative velocity of the fluid at the inflection point is zero. In the case of vortex shedding from the upper face, the streamline inflection occurs at the point of intersection of the lower face with the vortex wake. As a result, the relative velocity of the fluid at the lower corner point $v_{r\sigma}$, the intensity of vortices entering the wake γ_{wp}, the velocity of vortex shedding w_p, and the derivative $d\Gamma_p/dt$ are described by the relations

$$\begin{gathered} v_{r\sigma}(s_{*p}+0,t) = v_\sigma^+(s_{*p}+0,t) - U_\sigma(s_{*p},t) = \gamma(s_{*p}+0,t) = 0, \\ \gamma_{wp}(0,t) = \gamma(s_{*p}-0,t), \qquad w_p(t) = -\gamma(s_{*p}-0,t)/2, \\ \frac{d}{dt}\,\Gamma_p(t) = -\frac{1}{2}\,\gamma^2(s_{*p}-0,t) < 0. \end{gathered} \tag{3.5}$$

Similarly, vortex shedding from the lower face is described by the equations

$$\begin{gathered} v_{r\sigma}(s_{*p}-0,t) = v_\sigma^+(s_{*p}-0,t) - U_\sigma(s_{*p},t) = -\gamma(s_{*p}-0,t) = 0, \\ \gamma_{wp}(0,t) = \gamma(s_{*p}+0,t), \qquad w_p(t) = \gamma(s_{*p}+0,t)/2, \\ \frac{d}{dt}\,\Gamma_p(t) = \frac{1}{2}\,\gamma^2(s_{*p}+0,t) > 0. \end{gathered} \tag{3.6}$$

3.4. Smooth Contour. The point on a smooth contour may be considered as the limiting case of a corner point, with the only difference that the vortex-wake shedding point can move along the contour $L(t)$ with a velocity $\dot{s}_{*p}$. As a result, the transport velocity of the shedding point is

$$v_{e\sigma}(s_{*p},t) = \begin{cases} U_\sigma(s_{*p},t) + \dot{s}_{*p}(t), & d\Gamma_p/dt < 0, \\ U_\sigma(s_{*p},t) - \dot{s}_{*p}(t), & d\Gamma_p/dt > 0. \end{cases} \tag{3.7}$$

With allowance for Eq. (3.7), relations (3.5) and (3.6) for a smooth contour transform to the equations

$$\begin{gathered} \frac{d}{dt}\,\Gamma_p(t) < 0\colon \qquad v_{r\sigma}(s_{*p}+0,t) = \gamma(s_{*p}+0,t) - \dot{s}_{*p}(t) = 0, \\ \gamma_{wp}(0,t) = \gamma(s_{*p}-0,t) + \dot{s}_{*p}(t), \qquad w_p(t) = -[\gamma(s_{*p}-0,t) + \dot{s}_{*p}]/2, \\ \frac{d}{dt}\,\Gamma_p(t) > 0\colon \qquad v_{r\sigma}(s_{*p}-0,t) = \gamma(s_{*p}-0,t) + \dot{s}_{*p}(t) = 0, \\ \gamma_{wp}(0,t) = \gamma(s_{*p}+0,t) + \dot{s}_{*p}(t), \qquad w_p(t) = [\gamma(s_{*p}+0,t) + \dot{s}_{*p}]/2. \end{gathered}$$

Determining the arc coordinate $s_{*p}(t)$ of the point of vortex-wake shedding from a smooth contour is a difficult task. The basic criterion of vortex-wake shedding was proposed in [12]: it implies vanishing of the tangential component of the pressure gradient along the contour at the shedding point. For this condition to be valid, the arc coordinate of the shedding point should satisfy the equation

$$\begin{aligned} &\dot{U}_s(s_{*p},t) - \dot{\gamma}(s_{*p}-0,t) + [\gamma(s_{*p}-0,t) + \dot{s}_{*p}(t)]\,\frac{\partial}{\partial s}\,\gamma(s,t)\Big|_{s=s_{*p}-0} \\ &- [U_s(s_{*p},t) + \dot{s}_{*p}(t)]\,\frac{\partial}{\partial s}\,U_s(s,t)\Big|_{s=s_{*p}} - U_n(s_{*p},t)\,\frac{\partial}{\partial s}\,U_n(s,t)\Big|_{s=s_{*p}} = 0 \end{aligned} \tag{3.8}$$

for $d\Gamma_p/dt < 0$ or

$$\begin{aligned} &\dot{U}_s(s_{*p},t) - \dot{\gamma}(s_{*p}+0,t) + [\gamma(s_{*p}+0,t) + \dot{s}_{*p}(t)]\,\frac{\partial}{\partial s}\,\gamma(s,t)\Big|_{s=s_{*p}+0} \\ &- [U_s(s_{*p},t) + \dot{s}_{*p}(t)]\,\frac{\partial}{\partial s}\,U_s(s,t)\Big|_{s=s_{*p}} - U_n(s_{*p},t)\,\frac{\partial}{\partial s}\,U_n(s,t)\Big|_{s=s_{*p}} = 0 \end{aligned} \tag{3.9}$$

for $d\Gamma_p/dt > 0$.

Equations (3.8) and (3.9) are nonlinear differential equations of the first order with respect to $s_{*p}(t)$. The initial conditions are determined by setting the arc coordinates $s_{*p}(0)$ of the points on the contour L, where the hydrodynamic pressure reaches a local minimum in a steady flow and $\dot{s}_{*p}(0) = 0$. The functions $U_s(s,t)$, and $U_n(s,t)$ are prescribed, and the intensity of the vortex layer $\gamma(s,t)$ is determined by solving the corresponding nonlinear initial-boundary problem, which, in turn, depends on the position $s_{*p}(t)$ of flow-separation points and velocity of their motion $\dot{s}_{*p}(t)$. Thus, Eqs. (3.8) and (3.9) are only some of the nonlinear equations that are to be solved jointly for a particular initial-boundary problem of an unsteady separated flow around a smooth contour.

3.5. Kutta–Joukowski–Chaplygin Postulate. Shedding of the vortex wake from a sharp or angular edge ensures fulfillment of this postulate, because the fluid velocity at these edges remains finite during the entire time of motion of the contour. Rigorous satisfaction of conditions (3.5) and (3.6) at the corner point of the contour, however, leads to a paradox: the solution of the problem of an unsteady flow around a contour with an infinitesimal angle between the tangential lines at the corner point in the limiting case does not transform to the solution of the problem with a sharp edge (in the form of a cuspidal point) [10, 13]. This paradox is a consequence of restrictions of the ideal fluid model, which admits inflections of the streamlines in the neighborhood of the angular edge. Adaptation of this model by local replacement of an angular edge by a sharp edge yields physically meaningful results. It should be noted that such an adaptation is automatically included into calculation algorithms that do not require exact satisfaction of the condition of a zero relative velocity at the corner point on the streamline (e.g., if the vortex wake is modeled by a system of discrete vortices). A numerical experiment showed that local replacement of a corner point by a sharp point allows problems of the flow around contours of various shapes, including a circumference, to be effectively solved.

CONCLUSIONS

An attempt has been made to rigorously derive the governing equations and relations for a wide class of nonlinear initial-boundary problems of the wing theory in a two-dimensional unsteady flow of an ideal incompressible fluid. The results obtained make it possible to formulate particular problems and construct appropriate algorithms for solving them. Special attention is paid to an analysis of conditions of vortex-wake shedding from the contour, which are supplementary to the boundary and initial conditions. These conditions relate the intensities of vortex wakes to the intensities of the vortex layer on the airfoil contour, which allows closing the system of equations in the algorithm for solving a particular problem.

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